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ANALYTICAL APPROXIMATIONS

Volume 11

Cecil Hastings, Jr.

James P. Wong, Jr.

P-415

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1 July 1953

Approved for OTS release

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### Analytical Approximation

Bessel Function of Imaginary Argument: To better  
than .00007 over (0,2),

$$e^{-x} J_1(x) \doteq \frac{.4981x + .0066x^2}{1 + .9805x + .4477x^2} .$$

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### Analytical Approximation

Bessel Function of Imaginary Argument: To better than .000,006 over (0,1),

$$e^{-x}I_1(x) \doteq \frac{.49974x - .01695x^2}{1 + .95935x + .36282x^2}.$$

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# Analytical Approximation

Bessel Function of Imaginary Argument: To better  
than .0005 over  $(0, \infty)$ ,

$$e^{-x} I_1(x) \doteq \frac{x}{\sqrt{3.78 + 9.81x + 3.09x^2 + 6.36x^3}} .$$

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### Analytical Approximation

Mach Number in Terms of Pressure Ratio: To .001 over  
 $.3 \leq M \leq 1.0$  the inverse of

$$x = \frac{P_s}{P_A} = \left[ 1 + \left( \frac{\gamma-1}{2} \right) M^2 \right]^{-\frac{\gamma}{\gamma-1}},$$

where  $\gamma = 1.4$ , is given by

$$M \doteq \frac{2.714 - 2.625x}{1 + 1.650x - 1.955x^2}.$$

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Analytical Approximation

Bessel Function of Imaginary Argument: To better  
than .00005 over  $(2, \infty)$ ,

$$e^{-x} I_1(x) \doteq \frac{x}{\sqrt{10.281 + 3.752x + 4.541x^2 + 6.296x^3}}.$$

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